Lesson 1 – Relative Motion

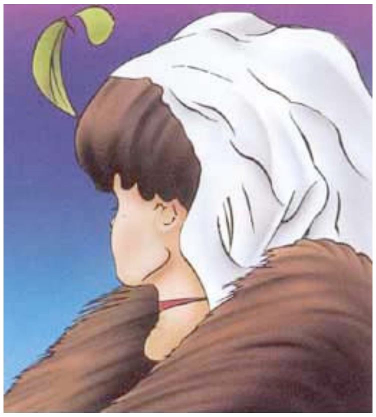
For many centuries, physicists thought that Newton's Laws of motion were absolute, in that they seemed to describe nature perfectly. But in 1905 Albert Einstein (who was 26 years old at the time) was able to demonstrate that Newton's second law became invalid at speeds close to that of light. Special relativity was born.

But before we can begin to study *special* relativity, we need to consider *plain old ordinary* relativity. This lesson will investigate just that, relative motion.

Consider the image to the right. What does the image depict?

Young woman?

Old lady?



Depending on your point of view, both answers are correct! Take another look. Each of you sees the world in a slightly different way – you all have your own point of view and this depends on your perception and past experience.

In physics, a **frame of reference** is

As it turns out all motion is relative. This is why we always must measure motion relative to a frame of reference. Consider the following rowboat example:

Both situations are mathematically and physically correct, and they serve as a perfect, simple example of one-dimensional relative motion. But it gets a bit more complex than this once we enter into two dimensions. We will use the following notation:

$\vec{v}_{\text{og}}$

$\vec{v}_{\text{mg}}$

$\vec{v}_{\text{om}}$

The best way to learn how all this fits together is to see a few examples of classic relative motion questions; the boat crossing a river, and air-navigation.

Example 1

George wants to cross the Grand River. He hops into his canoe in Kitchener and paddles straight north towards Waterloo with a velocity of 5.0 km/h. If the Grand River is 5.0 km wide, how long does it take him to reach the other side?

Example 2

On another day, George notices that there is a current flowing down the Grand River due east at 2.0 km/h towards Guelph.

a) How does the current affect the time required to cross the river?

b) How far is it from Waterloo to Guelph along the river's edge?

Example 3

On a third day of canoeing, George decides to head to Waterloo instead of Guelph. If the conditions are the exact same conditions as in Example 2:

a) Find the direction the boat must be pointed in order to land in Waterloo.
b) What is the ground velocity of the boat?
c) How long will it take the boat to reach the other shore?

Example 4

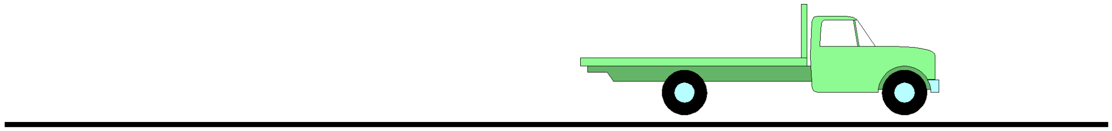
The BAU team from Criminal Minds needs to fly in the FBI private jet due north from Washington D.C. to Belleville, ON in order to profile an UnSub. There is a wind from the west at 20.0 km/h. If the plane can fly at a velocity of 150 km/h in still air:

a) what is the plane's heading (in which direction should the pilot point the plane)?
b) What is the plane's ground velocity?

Homework: Pg. 116 # 22, 23, 25, 28

Lesson 2 – Einstein's Postulates

Recall from our last lesson that all motion is relative depending on your frame of reference.



In the truck's frame of reference, the pitcher and catcher would agree on the speed of the baseball. As viewed from the ground however, an observer would see the baseball travel at a different speed.

The First Postulate

In general, there are two different types of reference frames:

Inertial Frame of Reference

Non-Inertial Frame of Reference

In thinking about these frames of reference, Einstein proposed a theory that boldly changed the way we look at space and time.

The Principle of Relativity

Consider now, an example of the truth of Einstein's first postulate.

Example 1

A physics student, Alphonse, is sitting at the dinner table when his 6.0 kg turkey suddenly explodes into two equal pieces. One piece moves 2.0 m/s [L], while the other moves 2.0 m/s [R]. At this very moment, Beauregard walks by the table moving at 2.0 m/s [L]. Determine the change in Kinetic Energy of the turkey from both A and B's point of view.

Example 1 *continued*

What makes this postulate so bold is that is asserts that **all** the laws of physics – those dealing with mechanics, electricity and magnetism, optics, thermodynamics, and so on – are the same in all reference frames moving with constant velocity relative to one another.

The Second Postulate

In grade 11, we learned that sound is a wave and travels at different velocities depending on the medium. We also saw evidence from the Doppler Effect that sound changes depending on certain conditions.

As we will learn in the coming units, light is a very unique subject. It has properties of both particles and waves. In fact, light is different than sound in many ways and a few revolutionary experiments taught us that light does not require a medium to propagate.

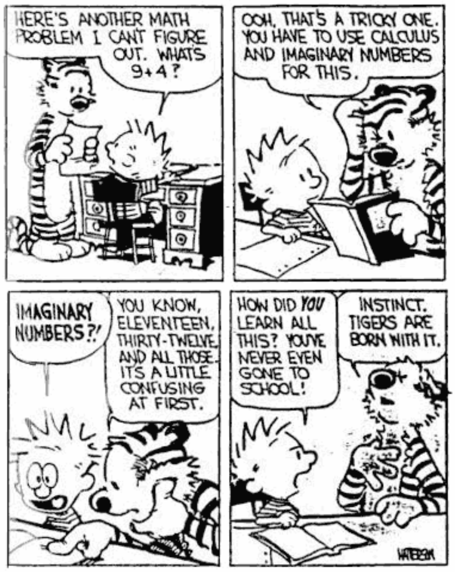
Consider this…

You and a friend are outside on opposite ends of the football field.
If your friend calls out to you, how might the sound change if there is a strong wind towards your friend?

Based on this and his conviction that the laws of physics are the same in all inertial frames of reference, Einstein developed his second postulate of special relativity.

The Constancy of the Speed of Light

Lesson 3 – Consequences of Special Relativity



If we are to accept the Einstein's theory of Special Relativity, we must conclude that relative motion (as we know it) is not important when measuring the speed of light. We also have to do away with our common sense notion of space and time. This can be difficult, especially considering that most (if not all) of our experience deals with objects moving no where near the speed of light. Get ready to experience the bizarre effects of moving close to the speed of light.

Time Dilation

Moving clocks run slow. This is one effect of the theory of special relativity. Using Einstein's two postulates and Pythagoras, we can show that the measurement of time depends on your speed.

Consider this… *Thought Experiment 1*

Alphonse is in a boxcar travelling at a speed v. He performs the following experiment: a pulse of light is transmitted from the floor up toward the ceiling, where it hits a mirror and is reflected back to the floor.

Beauregard, observing Alphonse's experiment from the ground as he rides by, notices something different than Alphonse. From Beauregard's point of view, the light travels up to the ceiling while the boxcar moves sideways. She notices the light travels farther than Alphonse does and she measures a time t greater than t_0.

If the laws of Physics must be the same for both A's and B's inertial frames of reference (first postulate) and light travels at c (second postulate) in both frames, A and B **must** measure a different time for the experiment!

Time, measured by Beauregard, is given by the equation

$$t = \frac{t_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where t is relativistic time, measured in a frame of reference where the beginning and end of the experiment occur *at two different points in space*; and t_o is proper time, measured in a frame of reference where the beginning and end of the experiment occur *at the same point in space.*

Example 1

The period of a pendulum is measured to be 3.0 s in the reference frame of the pendulum. What is the period when measured by an observer traveling at speed of 0.95c relative to the pendulum?

Length Contraction

Moving objects appear shorter. This is another affect of the special theory of relativity. As we noted above, time is relative, and using this relationship we can show that length measurements are also relative.

Consider this… *Thought Experiment 2*

Clyde is travelling on a spaceship from Earth to Mars and moving with a speed v. We will assume that Earth and Mars are at rest relative to each other and located a distance L_o apart.

For Delilah, an observer on Earth, the time taken for this trip is $t = \frac{L_o}{v}$. **Proper length**, L_o is the length measured in a reference frame in which the *observed object is at rest* (in this case, the distance between the planets).

For Clyde, proper time t_o (measured on the clock in his spaceship) is less because time dilates. Clyde sees himself at rest, with Mars approaching and Earth receding. Putting these things together we can calculate the **relativistic length** of Clyde's trip.

A general formula for relativistic length is given by

$$L = L_o\sqrt{1 - \frac{v^2}{c^2}}$$

Example 2

A spaceship is measured to be 120.0 m long while at rest relative to an observer. If the spaceship flies by the observer at 0.99c, what length does the observer measure?

The Twin Paradox

A paradox is a situation in which people reach contradictory conclusions using valid deductions from premises that are acceptable to everyone. The twin paradox is one of the most famous in relativity.

A set of twins named Eunice and Francisco reach an age of 20 years old. Eunice, who is more adventurous, sets out on an epic journey to the planet X, located 20 light-years from Earth. Furthermore, her spaceship is capable of reaching speeds of $0.95c$, relative to the inertial frame of her brother back home. After reaching the Planet X, Eunice becomes homesick and immediately returns to the Earth at the same speed $0.95c$. Upon her return, Eunice is shocked to discover that Francisco has aged 42 years and is now 62 years old. Eunice on the other hand, has only aged 13 years.

Example 3

Verify the final ages of the twins in the situation described above.

But here is the paradox: which twin was actually the traveller? From Francisco's frame of reference, it was he who stayed home while his sister Eunice travelled at a high speed. From Eunice's frame of reference, she was at rest while her brother was on the high-speed journey.

What is the problem with the Twin Paradox?